

University of Groningen

Predictions in Conjoint Choice Experiments

Haaïjer, Marinus E.; Vriens, Marco; Wansbeek, Tom J. ; Wedel, Michel

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version

Publisher's PDF, also known as Version of record

Publication date:

1996

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Haaïjer, M. E., Vriens, M., Wansbeek, T. J., & Wedel, M. (1996). *Predictions in Conjoint Choice Experiments: The X-Factor Probit Model*. University of Groningen, SOM research school.

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

The publication may also be distributed here under the terms of Article 25fa of the Dutch Copyright Act, indicated by the "Taverne" license. More information can be found on the University of Groningen website: <https://www.rug.nl/library/open-access/self-archiving-pure/taverne-amendment>.

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

Predictions in Conjoint Choice Experiments: The X-Factor Probit Model.

Marinus E. Haaijer

Marco Vriens

Tom J. Wansbeek

Michel Wedel

June 1996

SOM theme B: Inter-firm Interactions: Marketing and Networks

¹ M.E. Haaijer, University of Groningen, Faculty of Economics, Department of Marketing and Marketing Research. P.O. Box 800, 9700 AV Groningen, The Netherlands. Phone: +31-50-3637184, Fax: +31-50- 3637207, Email: M.E.Haaijer@ECO.RUG.NL. M. Vriens, Tilburg University, T.J. Wansbeek, University of Groningen, M. Wedel, University of Groningen.

Abstract

We develop a Multinomial Probit model with an X-factor covariance structure that can be used to estimate conjoint choice models. This model can be used for predictions which is not possible for general Multinomial Probit models. The model also solves problems related to the identification of the general Multinomial Probit model for Conjoint choice experiments. We show that in an application our model fits the data significantly better than the Multinomial Logit model and Independent Probit model. We assess the predictive validity of the model, and perform a Monte Carlo study that demonstrates that misspecified models (Multinomial Logit) may perform well on holdout predictions. However, we demonstrate that the substantive implications of predictions made with the misspecified Multinomial Logit model may differ importantly from predictions made with the X-factor Probit model.

1. Introduction

Experimental choice analysis is among the most frequently used methods for measuring and analyzing consumer preferences. Especially in the fields of product design and product optimization (Green and Krieger 1993) and the evaluation of competitive sets (Mahajan, Green and Goldberg 1982, Louviere and Woodworth 1983), experimental choice analysis has been successfully applied. In their recent JMR editorial on conjoint, Carroll and Green (1995) point out some of the advantages of conjoint choice models, among which that the task is more realistic and that it avoids the use of choice simulators. However, they also point at current gaps in our knowledge of experimental choice analysis, one of them being “How restrictive is the Independence of Irrelevant Alternatives Assumption of the Multinomial Logit model?” The purpose of the present study is to demonstrate that the IIA property may be too restrictive in conjoint choice experiments, and to illustrate that it may lead to incorrect insights.

Conjoint choice experiments are commonly estimated with Multinomial Logit (MNL) models (Louviere and Woodworth 1983, Elrod, Louviere and Davey 1992, Carroll and Green 1995). The latter model involves the Independence of Irrelevant Alternatives (IIA) property, resulting from the assumption that the error terms of the random utility of choice alternatives are uncorrelated, which may be a restrictive assumption in many situations (McFadden 1976, Green and Srinivasan 1978, Hausman and Wise 1978, Currim 1982, Maddala 1983, Kamakura and Srivastava 1984, McFadden 1986, Dalal and Klein 1988, Jain and Bass 1989, Carson et al 1994, Chintagunta and Honore 1996). Specifying a multivariate normal distribution of the error term of the random utility allows for a simple description of correlations among these error terms thereby avoiding the IIA property. However, the estimation of the resulting Multinomial Probit (MNP) models has been hampered by computational problems. When there are more than four profiles in a choice set, the integrals needed to obtain the choice probabilities can no longer be obtained numerically (e.g. Kamakura 1989). Simulation techniques have been developed to solve the integrals so that the estimation of MNP models now has become feasible through the method of simulated moments (e.g. McFadden 1989, Hajivassiliou 1993). The method of simulated moments has been used in marketing, for example,

by Chintagunta (1992) in a brand choice model and by Elrod and Keane (1996) a factor-analytic choice model, both applied to the analysis of scanner panel data.

Contrary to these previous application of simulation methods, we will use simulated likelihood to estimate an MNP model for conjoint choice experiments, and we will investigate the restrictiveness of the IIA property in such experiments. There are two serious complications when using a Multinomial Probit model in conjoint choice experiments. First, the unrestricted MNP model for choice experiments is in general not identified and therefore cannot be estimated consistently. Second, the general MNP model does not allow for predictions of holdout choice sets or new products, one of the most important applications of conjoint analysis. We will develop a particular specification of the covariance structure of the error term that alleviates these problems, allows for predictions of holdout choice sets and new products and that is identified. This leads to what we call the X-factor Multinomial Probit Model. In section 2 we discuss the Multinomial Logit and Probit models. In section 3 we provide the general Conjoint MNP model and we discuss briefly estimation by the simulated maximum likelihood technique. In section 4 we discuss the identification and prediction problems of MNP models for conjoint choice experiments and introduce the X-factor MNP model. In section 5 we present an application of an X-factor MNP model concerning the choice of coffee-makers. We compare the estimation results of the X-factor MNP model with those of the Independent Probit (IP) and MNL model. Furthermore, we provide a Monte Carlo study to compare the models on synthetic data. In section 6 we illustrate choice simulation results for competitive set analysis in three situations: product modification, product line extension and the introduction of a me-too brand (Green and Krieger 1993). We compare the X-factor MNP results to the MNL results and show that the latter may lead to incorrect conclusions. Conclusions are given in section 7.

2. Conjoint Choice Experiments

2.1 Experimental Choice Designs

In a conjoint choice experiment the respondents choose one alternative from each of several choice sets which have been constructed using experimental designs. The profiles are constructed from sets of attributes at certain numbers of levels by using factorial designs (Addelman 1962). Often a base alternative is included in all choice sets, which can also be a “none of those” alternative (Louviere 1988, Carso et al. 1994). In order to obtain part-worth utilities, choice models are estimated at the aggregate level.

The advantages of conjoint choice experiments, as compared to conventional conjoint analysis are that (1) there are no differences in scale between respondents, (2) choice situations may be more realistic than ranking or rating tasks, (3) respondents can evaluate more profiles, (4) choice probabilities can be directly estimated, and (5) one does not need ad hoc and potentially incorrect assumptions in order to create computerized choice simulators (Louviere 1988, Carroll and Green 1995). Disadvantages are that conjoint choice models are more complicated, that they require larger amounts of data and that the models cannot be estimated at the individual level, although latent class procedures have been developed that allow for estimation at the segment level (Kamakura, Wedel and Agrawal 1994, DeSarbo, Ramaswamy and Cohen 1995).

2.2 Multinomial Logit and Probit

Conjoint choice data are usually analyzed using the MNL model. The parameters of a MNL model, (as well as other random utility models), are usually estimated in the maximum likelihood framework (McFadden 1986). In the MNL model the choice probabilities have a closed algebraic form, which makes the estimation using maximum likelihood straightforward. However, the Independence of Irrelevant Alternatives (IIA) property, which arises from the assumption of

independent random errors, implies that the odds of choosing one alternative over another alternative must be constant regardless of whatever other alternatives are present (e.g. Louviere and Woodworth 1983, Ben-Akiva and Lerman 1985), which may often be too restrictive. If the IIA property holds, forecasting the choice probabilities of new alternatives can simply be done by inserting the attribute values of these new alternatives in the closed form expressions for the choice probabilities. These expressions may be expanded to accommodate ranking data, which is particularly useful in conjoint analysis (McFadden 1986, Kamakura, Wedel and Agrawal 1994). However, the assumptions needed to translate rankings into choice need not hold in practice, especially when individuals use elimination and nesting strategies, the IIA property does not hold (Louviere 1988). Also, the use of brand names in the conjoint design may result in correlations between the utilities of the alternatives, violating the IIA property. Green and Srinivasan (1978) stated that in consumer behavior contexts the IIA property may not be a realistic assumption especially when some of the alternatives are close substitutes (McFadden 1976). When covariances across alternatives are incorrectly assumed to be zero, the estimates for the effects of explanatory variables are inconsistent (Hausman and Wise 1978, Chintagunta 1992).

McFadden (1986) provided several ways to deal with problems arising from the IIA property. Most importantly, if the IIA property does not hold, other models which avoid IIA, should be used instead of the MNL model. However, at the cost of computational complexity. The most general of these models is the Multinomial Probit (MNP) model. The next section describes the MNP model as applied to conjoint choice experiments in more detail.

3. Conjoint MNP models

3.1 The basic model

In the conjoint experiment underlying our analysis, individuals choose one out of M alternatives, from each of K choice sets. These sets have no alternatives in

common, apart from one base-alternative that is present in each choice set. Hence, the total number of alternatives in the experiment equals $H=(M-1)K+1$. In our particular application (section 5) $H=17$, and we have two subsamples, one with $M=3$ and $K=8$ and one with $M=5$ and $K=4$. It is convenient to (arbitrarily) let the base-alternative be the first alternative. We make the usual assumption that each individual $i(i=1,\dots,I)$ chooses, from each choice set, the alternative with the highest utility. The (unobservable) utilities attributed to each of the H alternatives by individual i are contained in the H -vector $u_i=(u_{i1},\dots,u_{iH})'$, which is assumed to satisfy:

$$u_i = X\beta + e_i, \quad e_i \sim N_H(0, \Omega), \quad (1)$$

with $X=(x_1,\dots,x_H)'$ the $(H \times S)$ -matrix containing the attribute values and Ω the $(H \times H)$ covariance matrix of the error vector e_i .

Given this set-up, we can express the probability that individual i chooses alternative 1 from set 1, which contains alternatives 1, M . Let

$$A'_{11} = (-\mathbf{1}_{(M-1)}, I_{(M-1)}, 0_{(M-1) \times (H-M)})', \quad (2)$$

with $\mathbf{1}$ denoting a vector of ones. Then

$$\begin{aligned} \pi_{i1}(\beta, \Omega) &= P(u_{i1} > u_{i2}, \dots, u_{i1} > u_{iM}) \\ &= P(e_{i2} - e_{i1} < (x_1 - x_2)\beta, \dots, e_{iM} - e_{i1} < (x_1 - x_M)\beta) \\ &= \int_{-\infty}^{(x_1 - x_2)\beta} \dots \int_{-\infty}^{(x_1 - x_M)\beta} [(2\pi)^{M-1} |A'_{11} \Omega A_{11}|]^{-\frac{1}{2}} e^{-\frac{1}{2}(A'_{11} \Omega A_{11})^{-1} t} dt, \end{aligned} \quad (3)$$

Since X does not include individual specific characteristics in a conjoint experiment, this expression does not depend on i . The other probabilities can be expressed analogously.

When the choices from the different sets would be assumed to be independent observations, the log-likelihood is

$$\ln L = I \sum_{k=1}^K \sum_{j \in C_k} p_{kj} \ln(\pi_{kj}), \quad (4)$$

where C_k is the index set of the alternatives in choice set k and p_{kj} is the observed fraction of individuals choosing alternative j in set k . Maximizing the likelihood involves frequent evaluation of $(M-1)$ -dimensional integrals of the type as given in (3).

When we take into account that the same individuals have made a choice from

each of the K sets, the situation becomes more complicated. A simple example shows how. Let $K=2$ and $M=3$, hence $H=5$. So, there are two choice sets, with indexes $\{1,2,3\}$ and $\{1,4,5\}$. For each individual we observe two choices, one from each set. Consider an individual choosing “2” from set 1 and “4” from set 2. The resulting probability is:

$$P(u_{i2} > u_{i1}, u_{i2} > u_{i3}, u_{i4} > u_{i1}, u_{i4} > u_{i5}) . \quad (5)$$

This probability can be expressed as a function of β and Ω analogous to (3) involving a four-dimensional integral. When the two choices were made by different individuals we would have two two-dimensional integrals. In general, we now have to consider M^K different arrays containing the multiple choices, each corresponding with a probability involving an $(M-1)K$ -dimensional integral. In the empirical application with $M=3$ and $K=8$, or $M=5$ and $K=4$, the dimension is 16, which leads to an enormous computational burden even with simulation methods (see the next section). Hence we treat all choices as different observations and take the loss of certain amount of statistical efficiency in our estimations for granted. The resulting estimators for β and for the identified parameters in Ω (see section 4) are consistent but not efficient.

3.2 Estimation by Simulated Moments and Simulated Likelihood

Simulation techniques can be used to approximate the integrals in (3). The basic idea is to construct approximations for the probabilities by drawing repeatedly from the distribution of the error term in (1). McFadden (1989) and Pakes and Pollard (1989) introduced the Method of Simulated Moments (MSM). Later on, a variety of other simulators and simulation techniques have been developed. The simulators differ basically as to the way the drawing from the error distribution are obtained. Hajivassiliou, McFadden and Ruud (1993) compared the known simulators and concluded that the Smooth Recursive Conditioning (SRC) simulator, also known as the GHK-simulator (after Geweke, Hajivassiliou and Keane), is one of the best. Details about the SRC simulator can be found in Börsch-Supan and Hajivassiliou (1993), Hajivassiliou (1993), or Geweke, Keane and Runkle (1994).

The Method of Simulated Moments of McFadden has two drawbacks. First

all probabilities in the particular model have to be simulated and second, the estimators are only asymptotically efficient when optimal instruments are used (McFadden 1989, Hajivassiliou 1993). These optimal instruments depend on the true parameter values, which are generally unknown. With the method of Simulated Maximum Likelihood (SML), only the probabilities of the selected alternatives have to be simulated which is computationally more efficient. This method is known as a Smooth SML (SSML) when it is applied with a smooth choice simulator as the SRC-simulator. Estimates obtained with ML have well known statistical properties. With SSML asymptotic efficiency requires that $R/\sqrt{I} \rightarrow \infty$ as $I \rightarrow \infty$ (Börsch-Supa and Hajivassiliou 1993), where R is the number of simulations. However, several studies show that SSML is efficient in statistical sense even when the number of simulations is rather low, say 10 to 20 (Mühleisen 1991, Lee 1992, Börsch-Supa and Hajivassiliou 1993, Geweke, Keane and Runkle 1994). A potential drawback of using smooth simulators is that the simulated probabilities are not restricted to add up to one over the M choices (Mühleisen 1991, Lee 1992). However, according to Lee (1992), the adding-up property can always be satisfied by normalizing the original simulators, at the extra cost of simulating choice probabilities for all alternatives.

In most conjoint choice models, the X matrix does not depend on individual specific characteristics. Hence, only KM probabilities have to be simulated instead of IKM . This greatly lowers the computational burden, and makes the estimation of MNP models for conjoint choice experiments practically feasible.

4. Identification and prediction

When applying Conjoint MNP models to experimental choice analysis, two problems require attention, the problem of model identification and the problem of making (holdout) predictions. Both problems are tackled simultaneously by imposing a structure on Ω , as we will show below.

The first problem of the general MNP model that we address, is the inability to predict holdout profiles or to perform market simulations based on its parameters.

estimates. This unfortunate situation arises because in predicting choice probabilities for profiles not included in the design of the experiment, estimates of the covariances associated with these new profiles are required, which are not available. In the general Conjoint MNP model only (co)variances are estimated for profiles that constitute combinations of attributes levels actually present in the design. Thus the general MNP model specified in (1) cannot be used for prediction, which seriously limits its usefulness. Note that in the MNL and IP model all variances are equal, and all covariances are zero, which allows predictions to be made with these models.

Secondly, Bunch and Kitamura (1991) showed that Multinomial Probit models are often not identified. They demonstrated that nearly half of the published applications of MNP are based on non-identified models, which makes the interpretation of the estimates useless. In the MNP model with one choice set ($K=1$), only $M(M-1)/2-1$ of the $M(M+1)/2$ covariance parameters are identified (Dagsis 1985, Bunch 1991, Keane 1992). So, $M+1$ restrictions must be imposed. One solution is to set all covariance parameters of one alternative equal to zero. Also, the utility of one alternative can freely be set to zero, because in discrete choice model only differences in utilities are important and not absolute values (Keane 1992). However, even then the MNP model in general is not identified. Heckman and Sedlacek (1985) showed that a necessary condition for identification of the general MNP model is that the X-matrix contains at least one single regressor that varies across individuals. Keane (1992) showed that even when MNP models are formally identified, identification can still be "fragile" in the absence of so-called exclusion restrictions, implying that certain exogenous variables in the model do not affect the utility levels of certain alternatives. To avoid identification problems it is necessary to have one exclusion from each utility index (Keane 1992). These results extend directly to the situation of $K (>1)$ choice sets.

The first implication of the above for a Conjoint MNP model (with $K > 1$) are that only $KM(M-1)/2-1$ covariance parameters can be identified. Including a base alternative in the design is useful in that respect, because its covariances can be set equal to zero in all choice sets, and the model can be scaled by fixing one of the variances of the remaining alternatives in each choice set. Consequently, in each choice set, $M+1$ covariance parameters must be fixed for identification.

A second and even more restrictive implication of the above is that if all

individuals have the same design matrix X), a very common situation in Conjoint experiments, the Conjoint MNP model is not identified. There are two potential solutions to this problem. A first solution to the above problem is to include at least one variable in the model that varies across respondents and alternatives. This will result in a dramatic increase in the computational burden in estimating the model while such a variable may not be available. Second, one may use two, or more different design matrices, offered to different sub-samples of respondents. The number of probabilities that need to be simulated to estimate the model is now equal to KM times the total number of different designs that is used in the conjoint experiment. Also, additional combinations of profiles are introduced in the design which gives rise to new covariance parameters that need to be estimated, thereby raising the total number of parameters to estimate. Therefore, the number of different designs should be kept small, for example equal to two.

We propose a third solution, which simultaneously solves the prediction and identification problems in MNP models. We impose restrictions on the covariance matrix of the form:

$$\Omega = I + X\sigma\sigma'X', \quad (6)$$

where F is an $(S \times T)$ matrix of covariance parameters, I is the H -dimensional identity matrix and X is the $(H \times S)$ matrix representing attribute values, where H is the total number of different profiles in the model. The covariance structure (6) is a factor analytic structure, i.e. $\Omega = I + LL'$, with $L = XF$. Hence, we call this structure the X-factor MNP model. Instead of I , one may specify a general diagonal matrix containing profile specific variances, but predicting choices of new products becomes impossible again because their variances are unknown. Therefore, we will use the identity matrix I . In formulation (6), the covariances are expressed as a quadratic form in the attributes. This enables the calculation of the covariances of new products from the characteristics (the levels of the attributes) of the new product and the estimates for F . The number of covariance parameters that needs to be estimated in this MNP model is equal to ST . Often one may wish to set $T=1$ to obtain a sparse parameterization, since each additional column in F introduces S new parameters while additional restrictions need to be imposed for $T > 1$ to alleviate identification problems due to rotation freedom.

The X-factor model we propose can be interpreted as a random-taste variation

model (e.g. Hausman and Wise 1978, Ben-Akiva and Lerman 1985). Such a model is defined as:

$$u_i = X\beta_i + \epsilon_i, \quad \beta_i = \beta + \psi_i, \quad \text{with} \quad (7)$$

$$e_i \sim N_H(0, \Sigma_e), \quad \psi_i \sim N_H(0, \Sigma_\psi). \quad (8)$$

In our X-factor model we specify:

$$\Sigma_e = I, \quad \Sigma_\psi = \sigma\sigma', \quad \text{hence:} \quad (9)$$

$$\begin{aligned} e_i^* = X\psi_i + e_i &\sim N(0, \Sigma_e + X\Sigma_\psi X') \\ &\sim N(0, I + X\sigma\sigma'X'). \end{aligned} \quad (10)$$

Thus, the X-factor model captures consumer heterogeneity through a specific structure of the covariance matrix. We have extensively investigated the identification of the X-factor model by computational means, on synthetic and empirical data, by calculating the eigenvalues of the Hessian matrix in the optimum (the second order derivatives of the log-likelihood with respect to the parameters). The solution yielded positive eigenvalues in all cases studied, strongly suggesting that the model is identified (Bekker et al. 1994). We also have investigated identification of MNP models with other covariance structures, such as general one-factor structures $\Omega = I + LL'$ and $\Omega = D + LL'$, where D is a diagonal matrix and L a vector, as well as the most general, block-diagonal, structure. None of these models appeared to be identified as indicated by non-positive eigenvalues of the Hessian.

5. Application

5.1 Data

We provide an application of the proposed X-factor MNP model to the results of a conjoint choice study on coffee-makers. After in-depth discussions with experts and consumers, hypothetical coffee-makers were defined on five attributes: brand name, capacity, price, presence of a special filter and thermos-flask. The attributes and levels are listed in Table 1.

Table 1: Attributes and levels of coffee-makers

Attribute Level	Brand	Capacity	Price (Dfl)	Special Filter	Thermos-flask
1	Philips	6 cups	f 39,-	Yes	Yes
2	Braun	10 cups	f 69,-	No	No
3	Moulinex	15 cups	f 99,-		

A total of sixteen profiles was constructed, which was twice divided differently into eight sets of two alternatives and four sets of four alternatives. A base alternative was added to each set, resulting in eight choice sets with three alternatives and four choice sets with five alternatives for each of the two different replications, which were offered to two groups of respondents. Furthermore, eight holdout profiles were constructed; four holdout sets with three alternatives and two holdout sets with five alternatives, where the same base alternative was used as for the estimation data.

At a large shopping mall in the Netherlands, 185 respondents were recruited. These respondents were randomly divided into two groups of 94 and 91 subjects respectively. Each group was administered half of the constructed choice sets. So each respondent had to evaluate eight sets of three alternatives and four sets of five alternatives, as well as the holdout sets. (Note that the design of the study without two groups ensures identification).

The choices from the sets with three alternatives and the choices from the sets with five alternatives were modelled separately. Effects-type coding was used for the attributes dummies. We use $T=1$ in our X-factor model, i.e. a one-factor model. This model was identified in all cases studied, while $T=2$ model did not provide a significant improvement. For the simulations we use the SRC procedure of Hajivassiliou, McFadden and Ruud (1992) in the SML context, maximizing the log-likelihood of (4). We use 100 draws for the simulation of the probabilities. The optimization routine we use is the Broyden, Fletcher, Goldfarb and Shanno algorithm implemented in the Gauss-package (Aptech 1995).

For comparison we also estimated the IP and MNL model. The starting values for the parameters in the iterative optimization in the IP and MNL model were arbitrarily chosen to be zero, because the likelihood of the IP and MNL models has a unique maximum (Maddala 1983). We started the X-factor MNL model from the

IP solution. (In addition we also used 10 sets of random starting values for all parameters in the X-factor MNP model but these yielded virtually the same results which are not reported here).

5.2 Criteria for comparison

To compare the models we use five statistics. The first three, the log likelihood value, the AIC criterium (Akaike 1973) and the Pseudo R^2 value are calculated for overall comparison of the fit of the models. The Pseudo R^2 value is defined as $1 - \text{Ln}L / \text{Ln}L_0$ (Ben-Akiva and Lerman 1985), where $\text{Ln}L$ is the log-likelihood when all probabilities are equal to $1/M$. The AIC criterium is defined as $\text{AIC} = -2\text{Ln}L + 2n$, where n is the total number of estimated parameters in the model. The fourth statistic is the Z-value of Hausman and Wise (1978), which is defined for a Conjoint MNP model as:

$$Z = \sum_{i=1}^I \sum_{k=1}^K \sum_{m=1}^M \frac{(y_{ikm} - \hat{p}_{km})^2}{\hat{p}_{km}}, \quad (11)$$

where the \hat{p}_{km} s are the estimated probabilities in the IP and MNL model and the simulated probabilities in the X-factor MNP model. This statistic is used to compare the models at individual level. Finally, the fifth statistic we use is the χ^2 value of the actual and predicted number of choices in each choice set, where the latter is calculated as the (simulated) probability to choose a profile, times the number of respondents which had to choose from that choice set. Furthermore, we use the likelihood ratio test to compare the X-factor MNP model and the MNL model. Such a test cannot be used to compare the X-factor MNP model and the MNL model because these are not nested. However, the IP structure has the same characteristics as the MNL model and therefore yields similar results (Hausman and Wise 1978, Amemiya 1981). It is included to compare, indirectly, the results of the MNL model with those of the X-factor MNP model. The above described statistics were used to compare the three models both for the three- and five-alternatives estimation data, as well as for the three- and five-alternatives holdout data. Predictions of the holdout sets with three and five alternatives were made from the three- and five-alternatives estimation

results respectively.

5.3 Results

In Table 2 the parameter estimates of the X-factor MNP model, the IP and MNL model, and the statistics for model comparison are listed for the three alternatives estimation data and for the five-alternatives estimation data. First we inspect the results in the three-alternatives data. The LR-statistic for testing the I model versus the X-factor MNP model is significant ($LR=41.98$, $p<0.01$), indicating that the X-factor MNP model fits significantly better. The X-factor MNP model also has better values for the other statistics except for the χ^2 -statistic, which does not differ much between models, however.

The signs of the regression parameters are as expected for all three models: the lowest capacity and the highest price have a negative partial utility and the attributes Thermos-Flask and Special Filter have a positive partial utility when present. However, there are some differences in the magnitudes of the estimates across the three models. The X-factor MNP model has four significant covariance parameters, which is a clear indication that the IIA property does not hold for this data. This implies that the IP and MNL model estimates are inconsistent and not appropriate.

For the five-alternatives data, the estimates of the regression parameters are similar to those for the three-alternatives data. However, the parameters for Office are higher in the X-factor MNP model but have relative high standard errors which make these parameters insignificant. The estimates differ in magnitude across the three models. We again find significant covariance parameters in the X-factor MNP model which means that, also for choice sets with five alternatives, the IIA

Table 2: Estimation results

Model Attribute (level)	three-alternatives data			five-alternatives data		
	X-factor $\Omega=I+XFF'X'$	IP $\Omega=I$	MNL	X-factor $\Omega=I+XFF'X'$	IP $\Omega=I$	MNL
β_1 Brand (1)	-0.102 (.115)	0.014 (.045)	0.040 (.064)	-0.076 (.107)	0.019 (.045)	0.047 (.057)
β_2 Brand (2)	-0.184 * (.096)	-0.264 * (.045)	-0.329 * (.056)		-0.211 * (.041)	-0.283 * (.054)
β_3 Capacity (1)	-1.142 * (.140)	-0.778 * (.050)	-1.015 * (.065)	-0.157 * (.083)	-0.597 * (.048)	-0.917 * (.072)
β_4 Capacity (2)	0.577 * (.104)	0.372 * (.036)	0.493 * (.046)	-1.295 * (.281)	0.236 * (.037)	0.371 * (.049)
β_5 Price (1)	0.325 * (.119)	0.217 * (.057)	0.313 * (.069)	0.566 * (.118)	0.213 * (.041)	0.319 * (.052)
β_6 Price (2)	0.345 * (.105)	0.296 * (.044)	0.372 * (.057)	0.930 (.568)	0.098 * (.035)	0.134 * (.045)
β_7 Filter (1)	0.353 * (.062)	0.261 * (.031)	0.340 * (.040)	0.688 (.605)	0.209 * (.033)	0.293 * (.042)
β_8 Thermos (1)	0.271 * (.067)	0.244 * (.032)	0.312 * (.040)	0.248 * (.054)	0.273 * (.034)	0.378 * (.043)
				0.412 * (.075)		
σ_1 Brand (1)	0.702 * (.226)			0.484 * (.203)		
σ_2 Brand (2)	0.064 (.241)			0.190 (.210)		
σ_3 Capacity (1)	0.599 * (.225)			0.605 * (.183)		
σ_4 Capacity (2)	0.042 (.120)			-0.245 (.252)		
σ_5 Price (1)	-0.136 (.211)					
σ_6 Price (2)	0.481 * (.192)			1.459 (.910)		
σ_7 Filter (1)	0.283 * (.127)			1.177 (.731)		
σ_8 Thermos (1)	0.161 (.157)			0.317 * (.137)		
				-0.386 * (.118)		
Ln-Likelihood	-1278.901	-1299.891	-1298.706	-999.102	-1014.187	-1013.378
AIC	2589.802	2615.782	2613.412	2030.204	2044.374	2042.756
Pseudo R ²	0.213	0.201	0.201	0.161	0.148	0.149
Z	3061.461	2986.949	2974.172	3107.205	3018.335	2997.523
χ^2	20.247	59.381	57.883	24.778	56.491	53.827

*: p < 0.05

property does not hold. Note that the covariance parameters for the five-alternative data are somewhat different from those of the three-alternatives data. The X-factor MNP model scores best on almost all fit statistics. The LR test again indicates that the X-factor MNP model fits significantly better than the IP model (LR=30.17 $p<0.01$). The MNL and IP specifications yield similar values on all statistics for both cases, as expected.

Table 3: Eigenvalues Hessian matrix

E.V	three-alternatives data				five-alternatives data			
	X-factor		IP	MNL	X-factor		IP	MNL
λ_9/λ_9	0.002	0.121	0.116	0.063	0.001	0.190	0.334	0.135
λ_9/λ_{10}	0.006	0.164	0.179	0.095	0.010	0.276	0.509	0.234
λ_9/λ_{11}	0.009	0.208	0.302	0.164	0.026	0.398	0.579	0.273
λ_9/λ_{12}	0.016	0.251	0.446	0.253	0.039	0.567	1.111	0.516
λ_9/λ_{13}	0.020	0.291	0.663	0.430	0.071	0.688	1.230	0.659
λ_9/λ_{14}	0.033	0.483	0.721	0.454	0.089	0.856	1.458	0.731
λ_9/λ_{15}	0.051	0.616	0.858	0.558	0.129	1.121	1.531	0.820
λ_9/λ_{16}	0.058	0.933	1.937	1.121	0.140	1.358	2.126	1.040

Table 3 gives the eigenvalues of the Hessian matrices. It shows that all eigenvalues are strictly positive, indicating that all models estimated are identified which empirically supports the identification of our model. (For all models we estimate below it also holds that all eigenvalues are greater than zero, however we will not list these eigenvalues).

From Table 2 we can see that a number of covariance parameters in the X-factor MNP model have rather high values, but are not significant due to large standard errors. In using the X-factor MNP model for prediction, these large non significant covariances may influence the results. Therefore, we re-estimated the X factor MNP model for the three-alternatives data and for the five-alternatives data restricting the non-significant covariance parameters from Table 2 to zeros. The estimation results of the restricted X-factor MNP model are listed in Table 4.

Table 4: Estimation results restricted model

Model Attribute (level)	three-alternatives data	five-alternatives data
	X-factor $\Omega = I + X'FF'X'$	X-factor $\Omega = I + X'FF'X'$
β_1 Brand (1)	-0.039 (.066)	-0.049 (.070)
β_2 Brand (2)	-0.177* (.069)	-0.139 (.071)
β_3 Capacity (1)	-1.170* (.107)	-0.874* (.086)
β_4 Capacity (2)	0.600* (.082)	0.352* (.055)
β_5 Price (1)	0.486* (.192)	0.287* (.066)
β_6 Price (2)	0.313* (.070)	0.056 (.058)
β_7 Filter (1)	0.360* (.052)	0.245* (.048)
β_8 Thermos (1)	0.319* (.073)	0.371* (.056)
σ_1 Brand (1)	0.547* (.129)	0.635* (.176)
σ_2 Brand (2)	0	0
σ_3 Capacity (1)	0.519* (.165)	0.105 (.129)
σ_4 Capacity (2)	0	0
σ_5 Price (1)	0	0
σ_6 Price (2)	0.632* (.293)	0
σ_7 Filter (1)	0.246* (.116)	0.265* (.105)
σ_8 Thermos (1)	0	-0.373* (.155)
Ln-Likelihood	-1279.708	-1000.473
AIC	2583.416	2024.946
Pseudo R ²	0.213	0.160
Z	3087.445	3147.714
χ^2	22.110	28.052

*: p < 0.05

From Table 4 we can see that the estimates of the covariance parameters in the restricted X-factor MNP model for the three and five-alternatives data differ only slightly from those in Table 2 and are all significant for the three-alternatives data. Furthermore, the β -parameters are very similar to those in Table 2. The fit statistics in Table 2 are only slightly better than those in the restricted model of Table 4, so the restricted X-factor MNP model also fits the data significantly better (with respect to the likelihood) than the MNL and IP models.

The estimates of the models on the three and five-alternatives data were used to predict the holdout sets with three and five alternatives respectively. Table 5 gives the statistics for the three models for the prediction of choices in the four holdout choice sets with three alternatives and in the two holdout sets with five alternatives. Table 5 shows that none of the three models predicts the holdout sets with the three alternatives very well. The pseudo R² values are low for all three models. However,

the IP and MNL models appear to predict somewhat better than the X-factor MNP model.

Table 5: Fit of predictions holdout sets

Statistic	Holdout sets with three alternatives (estimates from three-alternatives data)			Holdout sets with five alternatives (estimates from five-alternatives data)		
	X-factor $\Omega = I + XFF'X$	IP $\Omega = I$	MNL	X-factor $\Omega = I + XFF'X$	IP $\Omega = I$	MNL
Ln-Likelihood	-781.671	-	-	-443.154	-	-
AIC	1587.342	759.354	754.858	910.308	448.434	447.675
Pseudo R^2	0.039	0.066	0.071	0.256	0.247	0.248
Z	3001.917	1906.18	1790.65	1672.203	1541.26	1560.93
χ^2	165.653	73.029	58.574	37.315	43.813	43.625

Table 5 also shows that the X-factor MNP model predicts the holdout set with five alternatives best (with respect to the likelihood). The LR-test indicates that this difference is significant (LR=10.56, $p < 0.05$). The Pseudo R^2 s are much higher for all three models than for the holdout sets with three alternatives.

The above analyses show that our X-factor MNP model performs significantly better on model fit compared to the MNL and IP models. However, the performance with respect to holdout predictions was only better in the situation of predicting holdout sets with five alternatives and not on predicting holdout sets with three alternatives. The results for fit and predictions seem contradictory, and in the next section we will perform a Monte Carlo analysis to further investigate these findings.

5.4 Monte Carlo Study

We used the parameters from Table 4 and the design from Table 1 to construct 50 synthetic data sets and 50 holdout sets with the X-factor MNP model. We use two groups each with 100 individuals. We both constructed synthetic data with three alternatives as well as with five alternatives. Each data set was estimated with all three models. The results for the MNP model are similar to those of the IP model in all situations, hence we will not discuss those separately.

In each of the 50 cases, for the three and five alternatives data, the log likelihood of the X-factor MNP model is significantly better than that of the IP model. Furthermore, the X-factor MNP model recovers the “true” parameters well. More informative is the predictive power of the three models when predicting the synthetic holdout sets that are based on the same underlying parameters. For the data with three alternatives we find that the X-factor MNP model predicts the holdout data in 44 of the 50 situations better than the IP model (LR-test, $p=0.05$). In 16 situations the likelihood is better, but not significantly and in the remaining 24 situations the likelihood is worse. For the data with five alternatives we find that the X-factor MNP model predicts the holdout data in only 10 of the 50 situations significantly better than the IP model. In 16 situations the likelihood is better, but not significantly and in the remaining 24 situations the likelihood is worse. The reason that the results of the X-factor MNP model are relatively better for the holdout data with three alternatives than for the holdout data with five alternatives is probably caused by the fact that there are less observations in the latter case, so the model estimates may be less stable.

The above analyses show that, even when the same “true” underlying MNP parameters are used to construct choice sets and holdout sets, the predictive power of a MNP model may be worse despite a significantly better fit. So, to state it the other way round, even a theoretically incorrect model, as the IP and MNL model in this situation, still may perform better on holdout predictions. This means that the conclusions drawn from the holdout predictions have limited validity and model fit is a better criterium to choose between models. When a model is accepted, its result can be used to make (holdout) predictions but these results should not be compared to (holdout) predictions of the rejected model. The substantive implications of predictions made with an incorrect (MNL, IP) model may be very different from those of the correct model (MNP) as we will show in the next section.

6. Choice simulations for new product introductions

Here we illustrate that assuming incorrectly that the IIA property holds lead

to substantially different predictions of market-shares with the MNL as compared to the X-factor MNP model. We consider three managerially relevant situations: product line extension, a product modification and the introduction of a new brand. In our simple hypothetical illustrations we use the four profiles listed in Table 6.

Table 6: Attributes of prediction profiles

Attribute Profile	Brand	Capacity	Price (Dfl)	Special Filter	Thermos- flask
1	Philips	10 cups	f 69,-	No	Yes
2	Braun	15 cups	f 69,-	Yes	Yes
3	Moulinex	15 cups	f 39,-	Yes	No
4	Philips	10 cups	f 69,-	Yes	No

In the analysis we will only compare the MNL with the X-factor MNP model, using the estimates obtained from the three-alternatives data from Table 4. Note that the MNL predictions are obtained from closed form expressions, while those of the MNP model are obtained by simulation. First consider the situation of a product line extension of the brand Philips. We assume that the current market consists of two products: Moulinex (profile 3) and Philips (profile 4). Assume further that Philips introduces an extension of its product-line represented by a new product (profile 1) that only differs from the existing product (profile 4) in that it has a thermos-flask instead of a special filter. The probabilities predicted with the MNL and X-factor MNP model, before and after the product-line extension, are provided in Table 7.

From Table 7 it can be seen that both models predict almost the same probabilities in the initial situation with two products. However, after the product line extension, the MNL model predicts that the market-share of Moulinex drops from 55.5% to 39.0% (a drop of 16.5%). The X-factor MNP model, however, predicts only a decrease of the market-share of Moulinex of 6.5% (from 54.6% to 48.1%). The X-factor MNP model predicts a lower market-share for the new Philips product as compared to the MNL model (20.9% versus 29.6%), and it predicts that this market-share is drawn relatively more from the existing product of Philips. This difference between the two models is caused by the IIA property of the MNL

Table 7: X-factor and MNL Predictions

	Product-line extension			Product modification			Intro. “me-too” brand		
	Brand	X-factor	MNL	Brand	X-factor	MNL	Brand	X-factor	MNL
Before	M (3)	.5457	.5546	P (1)	.2501	.2688	P (1)	.4285	.4317
	P (4)	.4543	.4454	B (2)	.3190	.3774	M (3)	.5715	.5683
				M (3)	.4308	.3538			
After	M (3)	.4807	.3902	P (4)	.3028	.2798	P (1)	.2501	.2688
	P (4)	.3108	.3134	B (2)	.2786	.3717	M (3)	.4308	.3538
	P (1)	.2085	.2964	M (3)	.4185	.3484	B (2)	.3190	.3774

model: when two alternatives are closely related (as are the two products of the same brand), the MNL model predicts a too high joint probability (market-share) of these two alternatives (0.61 versus 0.52 for X-factor MNP) (Green and Srinivasan 1978).

The second hypothetical example pertains to a product modification. In the initial situation three brands are in the market: Philips, Braun and Moulinex (Table 7). The probabilities predicted by the two models in this initial situation are already somewhat different. The X-factor MNP model predicts that Moulinex is market leader, while the MNL model predicts Braun to be market leader. When the first brand (Philips) modifies its product (from profile 1 to profile 4), the MNL model predicts an increase in market-share of only 1.1%, drawn proportional from Braun and Moulinex. The X-factor MNP model, however, predicts an increase of 5.3% in market-share drawn from the second brand Braun. In the MNL model Philips remains the smallest brand, while it becomes second in the X-factor MNP model. The MNL model predicts that the market structures will hardly be influenced by the product modification, while it does change under the MNP model.

A third situation considered here is the introduction of a new brand with relatively similar characteristics as (one of) the brands already in the market (a me too brand). Consider the situation of two brands in the market (Philips and Moulinex). Now a third brand, Braun, introduces a coffee-machine close to the existing product of the initial market-leader Moulinex, but with one feature less and at a lower price. Table 7 gives the market-shares predicted by the X-factor MNP and MNL models, before and after the introduction. Table 7 shows that the predictions of the models after the introduction are quite different, while the predictions in the initial situation were relatively similar. After the introduction, the MNL model predicts the highest market-share for the new brand (Braun), whereas the X-factor MNP model still predicts the highest market-share for the initial market-leader Moulinex, which is more intuitive. The X-factor MNP model predicts that the market-leader loses 14.1% market-share as a result of the introduction of the “me too” brand, while the MNL model predicts it to lose 21.5%. In either case Philips loses about 18% market-share as a consequence of the introduction of Braun. Note that in this situation the MNL model incorrectly predicts that the me-too brand (Braun) becomes the market-leader, while the X-factor MNP model predicts that Moulinex remains the market-leader.

7. Conclusion and discussion

In this paper we have shown that a X-factor MNP model is appropriate for the analysis of conjoint choice experiments. We have shown how an MNP model can be specified such that two problems of the MNP model, i.e. that it is not identified (for conjoint choice data) and that it cannot be used for predictions, are dealt with. Given the identification problems of MNP models, we recommend that in applications of MNP models to conjoint choice experiments identification of the model should be thoroughly investigated, by checking the eigenvalues of the Hessian.

The proposed X-factor MNP model does not suffer from the very restrictive IIA property of the Multinomial Logit and Independent Probit model. In the application we showed that the X-factor MNP model leads to a significantly better fit than the MNL model. Significant covariance parameters were found indicating that in the application, the IIA property did not hold. However, the MNL and IP models predicted holdout sets better than the X-factor MNP model, in one of two occasions.

A Monte Carlo study showed that when the same underlying parameters are used in estimation data and holdout data, the X-factor MNP model fits the estimation data significantly better as the MNP and IP model, but may not predict holdout sets better as those models. Further research should address the predictive validity of the proposed MNP model in general, the usefulness of holdout tasks in this respect and the predictive validity in relation to choice set complexity.

We have shown that the market-shares predicted with the X-factor MNP model in a number of relevant, hypothetical, situations differ from those obtained from the MNL model. The MNL specification clearly suffers from the IIA property when predicting the effects of products being introduced that are close to existing products in the market. This situation may occur either in the case of product line extensions or in the case of the entrance of a me-too brand into the market. However, also in the situation of a product modification, we showed that market-share predictions made with the MNL model may differ substantially from predictions made with the X-factor MNP model.

References

- Addelman, S. (1962), "Orthogonal Main-Effects Plans for Asymmetrical Factorial Experiments," *Technometrics*, 4, February, 21-46.
- Akaike, H. (1973), "Information Theory and an Extension of the Maximum Likelihood Principle," in B.N. Petrov and F. Csáki, eds, 2nd International Symposium on Information Theory, Akadémiai Kiadó, Budapest, 267-281.
- Amemiya, T. (1981), "Qualitative Response Models: A Survey," *Journal of Economic Literature*, 19, December, 1483-1536.
- Aptech (1995), "Gauss Applications," Aptech Systems, Inc. Maple Valley
- Bekker, P.A., A. Merckens and T.J. Wansbeek (1994), "Identification, Equivalent Models, and Computer Algebra," Academic Press, San Diego.
- Ben-Akiva, M. and S.R. Lerman (1985), "Discrete Choice Analysis: Theory and Application to Travel Demand," MIT Press.
- Börsch-Supan, A. and V.A. Hajivassiliou (1993), "Smooth Unbiased Multivariate Probability Simulators for Maximum Likelihood Estimation of Limited Dependent Variable Models," *Journal of Econometrics*, 58, 3, 347-368.
- Bunch, D.S. (1991), "Estimability in the Multinomial Probit Model," *Transportation Research B*, 25B, 1, 1-12.
- and Kitamura (1991), "Multinomial Probit Model Estimation Revisited: Testing Estimable Model Specifications, Maximum Likelihood Algorithms, and Probit Integral Approximations for Trinomial Models Of Household Car Ownership," Working paper, University of California.
- Carroll, J.D. and P.E. Green (1995), "Psychometric Methods in Marketing Research: Part 1, Conjoint Analysis," *Journal of Marketing Research*, XXXII, November, 385-391.
- Carson, R.T., J.J. Louviere, D.A. Anderson, P. Arabie, D. Bunch, D.A. Hensher, R.M. Johnson, W.F. Kuhfeld, D. Steinberg, J. Swait, H. Timmermans, and J.B. Wiley (1994), "Experimental Analysis of Choice," *Marketing Letters*, 5:4, 351-368.
- Chintagunta, P.K. (1992), "Estimating A Multinomial Probit Model Of Brand Choice Using The Method Of Simulated Moments," *Marketing Science*, 11, 4,

386-407.

- Chintagunta, P.K. and B.E. Honore (1996), "Investigating the Effects of Market Variables and Unobserved Heterogeneity in a Multinomial Probit Model, *International Journal of Research in Marketing*, 13, 1, 1-15.
- Currim, I.S. (1982), "Predictive Testing of Consumer Choice Models Not Subject to Independence of Irrelevant Alternatives," *Journal of Marketing Research*, 19, 2, 208-22.
- Dalal, S.R. and R.W. Klein (1988), "A Flexible Class of Discrete Choice Models, *Marketing Science*, 7, 3, 232-251.
- Dansie, B.R. (1985), "Parameter Estimability in the Multinomial Probit Model, *Transportation Research B*, 19B, 6, 526-528.
- DeSarbo, W.S., V. Ramaswamy and S.H. Cohen (1995), "Market Segmentation with Choice-Based Conjoint Analysis, *Marketing Letters*, 6, 2, 137-147.
- Elrod, T. and M.P. Keane (1995), "A Factor-Analytic Probit Model for Representing the Market Structures in Panel Data, *Journal of Marketing Research*, 32, 1, 1-16.
- Elrod, T., J.J. Louviere and K.S. Davey (1992), "An Empirical Comparison of Rating-Based and Choice-Based Conjoint Models," *Journal of Marketing Research*, XXIX, August, 368-377.
- Geweke, J., M.P. Keane and D. Runkle (1994), "Alternative Computational Approaches to Inference in the Multinomial Probit Model, *Review of Economics and Statistics*, 76, 4, 609-632.
- Green, P.E. and A.M. Krieger (1993), "Conjoint Analysis with Product-Positioning Applications," In *Handbooks in Operations Research and Management Science*, J. Eliashberg and G.L. Lilien (Eds.), Vol. 5. Marketing
- and V. Srinivasan (1978), "Conjoint Analysis in Consumer Research: Issues and Outlook," *Journal of Consumer Research*, 5, September, 103-123.
- Hajivassiliou, V.A. (1993), "Simulation Estimation Methods for Limited Dependent Variable Models," In *Handbook of Statistics*, G.S. Maddala, C.R. Rao and H.D. Vinod (Eds.), Vol. 11, 519-543.
- , D. McFadden and P. Ruud (1993), "Simulation of Multivariate Normal Rectangle Probabilities and their Derivatives: Theoretical and Computational Results," Working paper, Yale University.

- Hausman, J.A. and D.A. Wise (1978), "A Conditional Probit Model for Qualitative Choice: Discrete Decisions Recognizing Interdependence and Heterogeneous Preferences," *Econometrica*, 46, 2, 403-426.
- Heckman, J.J. and G. Sedlacek (1985), "Heterogeneity, Aggregation, and Market Wage Function: An Empirical Model of Self-Selection in the Labor Market," *Journal of Political Economy*, 93, 6, 1077-1125.
- Jain, D.C. and F.M. Bass (1989), "Effect of Choice Set Size on Choice Probabilities: An Extended Logit Model," *International Journal of Research in Marketing*, 6, 1, 1-11.
- Kamakura, W.A. (1989), "The Estimation of Multinomial Probit Models: A New Calibration Algorithm," *Transportation Science*, 23, 4, 253-265.
- and R.K. Srivastava (1984), "Predicting Choice Shares Under Conditions of Brand Interdependence," *Journal of Marketing Research*, 21, November, 420-434.
- , M. Wedel and J. Agrawal (1994), "Concomitant Variable Latent Class Model for Conjoint Analysis," *International Journal of Research in Marketing*, 11, 5, 451-464.
- Keane, M.P. (1992), "A Note on Identification in the Multinomial Probit Model," *Journal of Business & Economic Statistics*, 10, 2, 193-200.
- Lee, L.-F. (1992), "On Efficiency of Methods of Simulated Moments and Maximum Simulated Likelihood Estimation of Discrete Response Models," *Econometric Theory*, 8, 518-552.
- Louviere, J.J. (1988), "Conjoint Analysis Modelling of Stated Preferences: A Review of Theory, Methods, Recent Developments and External Validity," *Journal of Transport Economics and Policy*, January, 93-119.
- and G. Woodworth (1983), "Design and Analysis of Simulated Consumer Choice or Allocation Experiments: An Approach Based on Aggregate Data," *Journal of Marketing Research*, 20, 4, 350-367.
- Maddala, G.S. (1983), "Limited-Dependent and Qualitative Variables in Econometrics," Cambridge University Press.
- Mahajan, V., P.E. Green and S.M. Goldberg (1982), "A Conjoint Model for Measuring Self and Cross-Price/Demand Relationships," *Journal of Marketing Research*, XIX, August, 334-342.

- McFadden, D. (1976), "Quantal Choice Analysis: A Survey," *Annals of Economic and Social Measurement*, 5, 4, 363-390.
- (1986), "The Choice Theory Approach to Market Research," *Marketing Science*, 5, 4, 275-297.
- (1989), "A Method of Simulated Moments for Estimation of Discrete Response Models Without Numerical Integration," *Econometrica*, 57, 5, 995-1026.
- Mühleisen, M. (1991), "On The Use of Simulated Estimators for Panel Models with Limited Dependent Variables," Working paper, University of Munich.
- Pakes, A. and D. Pollard (1989), "Simulation and the Asymptotics of Optimization Estimators," *Econometrica*, 57, 5, 1027-1057.